NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

SCHOOL OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION : Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BAMS	**LEVEL: 5**
COURSE: PROBABILITY THEORY 1	**COURSE CODE:** PBT501S
DATE: JULY 2019	
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr. D. Ntirampeba
MODERATOR:	Mr. A. Roux

THIS QUESTION PAPER CONSISTS OF 5 PAGES
(Including this front page)

INSTRUCTIONS

1. Answer ALL the questions.
2. Write clearly and neatly.
3. Number the answers clearly.

PERMISSIBLE MATERIALS

Calculator.

ATTACHMENTS

Statistical tables (Z-TABLE).

Question [25 marks]

1.1. Define the following terminologies as they apply to probability theory

1.1.1. A random experiment. [2]

1.1.2. An event. [2]

1.1.3. A partition of a sample space S. [2]

1.2. Indicate which of the following random variables are **d = discrete**, and which are **c = continuous**:

1.2.1. The time required to answer this question. [1]

1.2.2. The number of words in a book chosen at random from the library. [1]

1.2.3. The direction of the wind (measured in degree) recorded at a weather station. [1]

1.2.4. The maximum temperature recorded at Hosea Kutâko Airport. [1]

1.3. Write down the ***correct word or statement*** which completes each sentence. [10X1]

1.3.1. A sample space is defined as _____.

1.3.2. What is the probability of an impossible event? _____.

1.3.3. A 6-sided die is rolled once. What is the probability that the number obtained is greater than 4? _____.

1.3.4. The probability P(A/B) is read in words as_____.

1.3.5. If two events A and B in a sample space are mutually exclusive then

$P(A \cup B) =$ _____.

1.3.6. If two events A and B in a sample space are mutually not exclusive then

$P(A \cup B) =$ _____.

1.3.7. In a multiple choice test with four possible answers for each question, what is the probability of answering a question correctly if you make a random guess? Note that only one answer is correct for each question.

1.3.8. If two events A and B in a sample space are independent then

$P(A \cap B) =$ _____.

1.3.9. In a sample space $S = \{1,2,3,4,5,6\}$, are the events $A = \{2,4,6\}$ and $B = \{4,5,6\}$ mutually exclusive?_____ Explain why.

1.3.10. In a sample space $S = \{1,2,3,4,5,6\}$, are the events $A = \{2,4,6\}$ and $B = \{4,5,6\}$ collectively exhaustive?_____ Explain why.

1.4. Suppose that a class contains 15 boys and 30 girls, and that 10 students are to be selected at random for a special assignment. What is the probability that exactly 3 boys will be selected? [5]

Question2 [25 marks]

2.1. The probability of surviving a certain transplant operation is 0.5. If a patient survives the operation, the probability that his or her body will reject the transplant within a month is 0.2. What is the probability of surviving both critical stages? [3]

2.2. A fast-food restaurant chain has 600 outlets in United States. The following table categorizes cities by size and location, and presents the number of restaurants in the cities of each category. A restaurant is to be chosen at random from 600 to test market a new style of chicken.

Population of city	Region			
	NE	SE	SW	NW
Under 50000	30	35	15	5
50000-500000	60	90	70	30
Over 500000	150	25	30	60

2.2.1. What is the probability the restaurant is in a city with a population under 50000 and is located in the Northeast? [3]

2.2.2. What is the probability the restaurant is in a city with a population over 50000 or is located in the Northwest? [4]

2.2.3. If the restaurant is located in a city with a population over 500000, what is the probability that it is in the Northeast? [4]

2.2.4. If the restaurant is located in South (either SE or SW), what is the probability that it is in a city with a population of 50000 or more? [4]

2.3. In a certain city, 30 percent of the people are Conservatives, 50 percent are Liberals, and 20 percent are Independents. Records show that in a particular election, 65 percent of conservatives voted, 82 percent of Liberals voted, and 50 percent of the independent voted. If a person in the city is selected at random and it is learned that he did not vote in the last election, what is the probability that he is a Liberal? [7]

Question 3 [25 marks]

3.1. Assume that $Y_1, Y_2, and\ Y_3$ are independent random variables, with

$$E(Y_1)=2 \qquad E(Y_2)=1 \qquad E(Y_3)=-4$$
$$V(Y_1)=4 \qquad V(Y_2)=3 \qquad V(Y_3)=6$$
Let $U=3Y_1-6Y_2-4Y_3$

Find:

3.1.1. Find $E(U)$ [3]

3.1.2. Find $V(U)$ [3]

3.2. The number of residential homes (N) that a fire can serve depends on the distance r (in city blocks) that a fire engine can cover in a specified (fixed) period of time. If we assume N is proportional to the area of a circle of R blocks from the firehouse, then $N = C\pi R^2$, where C is a constant, $\pi = 3.1416$, and R, a random variable, is the number of blocks that a fire engine can move in the specified time interval. For a particular fire company, $C = 8$, the probability distribution for R is as shown in the table below.

r	21	22	23	24	25	26
$p(r)$	0.05	0.2	0.3	0.25	0.15	0.05

3.2.1. Find the expected value of N, the number of homes that the fire department can serve. [5]

3.2.2. Find the variance of N, the number of homes that the fire department can serve. [6]

3.2.3. Find the coefficient of variation for N, the number of homes that the fire department can serve. [3]

3.3. A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Use Chebyshev's theorem to estimate $P(|X - 10| \geq 3)$ [5]

QUESTION 4 [25 marks]

4.1. Major software manufacturers offer a help line that allows customers to call and receive assistance in solving their problems. However, because of the volume of calls, customers frequently are put on hold. One software manufacturer claims that only 20% of callers are put on hold. Suppose 15 customers call,

4.1.1. find the probability that no more than two customers are put on. [4]

4.1.2. what is the expected number of callers that will be put on hold? [2]

4.2. The demand for daily newspaper at newsstand at a busy intersection is known to be normally distributed with mean of 850 and a standard deviation of 25. How many newspapers should the newsstand operator order to ensure that he runs short no more than 20% of days? [5]

4.3. Research has shown that 12 students call the NUST library helpdesk every 30 minutes. What is the probability that at most 5 students call the NUST library helpdesk in the next 10 minutes time? [5]

4.4. Consider a probability mass function of X given by

$$P(x) = \begin{cases} \binom{3}{x} 0.5^3 \ , & x = 0,1,2,3 \\ 0, & elsewhere \end{cases}$$

4.4.1. Find, $F(x)$, the distribution function of the random variable X [7]

4.4.2. Find the median of X [2]

END OF EXAM PAPER

Standard Normal Probabilities

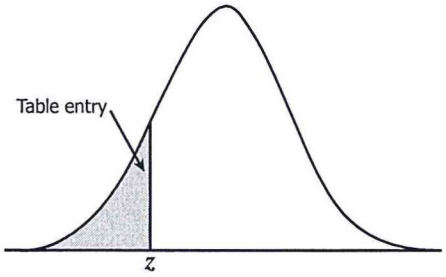


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

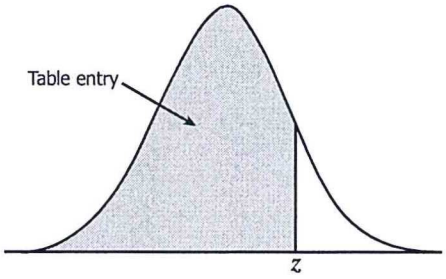
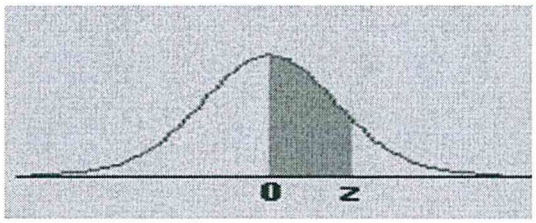


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

The Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990